Minimum Probability of Loss Trading Strategy for Mean Reverting Portfolios

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ABSTRACT
In this paper we propose a novel trading strategy and portfolio selection which aims at minimizing the loss probability based on identifying mean reverting portfolios. After obeserving historical data for parameter identification, the portfolio selection is performed by minimizing the probability of negative return (loss). The performance of the method have been tested by extensive performance analysis on FOREX historical time series. The proposed trading algorithm has achieved 5% percent yearly return on the average which proves its applicability to algorithmic trading.

Keywords – Mean Reversion, Algorithmic trading, Portfolio optimization, FOREX.

1 Introduction

Portfolio optimization was first investigated by Markowitz (Markowitz, 1952)in the context of diversification to minimize the associated risk and maximize predictability. Since the first results, many papers have been dealing with portfolio optimization, e.g. (Manzan, 2007) (J.W., 2002) (D'Aspremont, 2011) (Banerjee, O., El Ghaoui, L., d'Aspermont A., 2008) (R Balvers, 2000) (Grant, 2010). One of the usual approaches is finding the portfolio which exhibits predictability and minimal risk (D'Aspremont, 2011).Another approach is to identify mean reverting portfolios, where trading actions (e.g. buying orr selling) are launched when being out of the mean and a complementer actions are taken (e.g. selling or buying) after reverting to the mean (J.W., 2002). The traditional strategies, are concerned with optimizing the mean reverting parameter of predictability which lends itself to analytical tractability by solving an eigenvalue problem (D'Aspremont, 2011). In this paper we assume that the multidimensional asset price vector out of which the portfolio is to be constructed can be modeled by a VAR(1) process. Then portfolio selection can be broken down in two steps: (i) fast model identification based on the previously observed samples of the corresponding asset prices and; (ii) choosing a portfolio vector which minimizes the probability of negative return.

The paper treats these ideas in the following structure:

- In Section 2, the model and notations are introduced together with the mean reverting model. The system parameters are also estimated here.
- In Section 3, we deal with portfolio selection subject to minimizing the probability of loss. The trading strategy is is detailed in this section
- in Section 4, an extensive performance analysis of the new method compared to traditional (mean reverting strategies) and the corresponding numerical results are given;
- in Section 5, conclusions are drawn.

2 System model

In this section we describe the system model, which is used throughout the paper.

2.1 Parameters of assets, and investors

The sell price of a given asset i at a given time instant t is denoted by $s_i(t)$ and the buy price of the same asset is $b_i(t),\ b_i(t) > s_i(t)$ where $b_i(t) - s_i(t)$ is the so-called bid-ask spread. Assuming that a given number of assets $i = 1,.,N$ is available for trading, we denote the corresponding bid and ask prices by vectors

$$(\underline{b}(t), \underline{s}(t)).$$

respectively. We always use $\underline{x}$ as a column vector, and $\underline{x}'$ denotes the row vector being the transposed version of $\underline{x}$. An investor is supposed to have a linear combination of assets where the numbers of different indvidual assets at hand are denoted by vector$\underline{n}(t)$ (which is a sparse integer vector including many zero components for the sake of low transaction cost). Vector $\underline{n}(t)$ represents the portfolio. It is noteworthy, that negative components in this vector are also permitted, which refer to short positions. Additionally, the available cash of the investor is denoted by $c(t)$. As a result, the state of the investor can be fully described by a vector which is time dependent:

$$(\underline{n}(t), c(t)).$$

The current value of assets depends on the sell price $\underline{s}(t)$if the amount of assets are positive (long position), or on the buy price $\underline{b}(t)$ if the amount of assets are negative (short position). As a result, the value of the portfolio held by the investor is given as

$$P(t) = \max(\underline{0}',\ \underline{n}'(t))\ \underline{s}(t) + \min(\underline{0}',\ \underline{n}'(t))\ \underline{b}(t), \qquad (1)$$

where vector $\underline{0}$ refers to the all-zero vector (each component is zero)and the minimum and maximum operations are meant componentwise.

The wealth of the investor (or the available cash after selling the portfolio) at time instant t is

$$r(t) = P(t) + c(t).$$

This expression describes the amount of money the investor could obtain immediately, if he/she decides to close his/her positions (quitting from trading).

2.2 Description of mean reverting portfolios

In stock markets, the tendencies in the price of a given asset or portfolio is hard to describe and predict. However, there are some properties, which can make it possible to see some dominant rules of the price movements. Here, we will focus on mean reverting portfolios, especially sparse portfolios (in which only a few number of assets are used for making the portfolio to minimize the transaction cost). Mean reverting portfolios can be described by the Ornstein-Uhlenbeck stochastic differential equation

$$dP(t) = \lambda(\mu - P(t))dt + \sigma\, dW(t), \qquad (2)$$

where $P(t)$ is the value of the portfolio, defined in portfolio, λ, μ, σ are parameters, which can be estimated based on observations,

Following the construction of d'Aspremont, We view the asset prices as a stationary, first order, vector autoregressive VAR(1) process. Let $s_{i,t}$ denote the price of asset *i* at time instant *t*, assume that $\mathbf{s}_t^T = (s_{1,t}, ..., s_{n,t})$ is subject to a first order vector autoregressive process, VAR(1), defined as follows:

$$\mathbf{s}_t = \mathbf{A}\mathbf{s}_{t-1} + \mathbf{W}_t,$$

where**A** is an *nxn* matrix and $\mathbf{W}_t \sim N(0, \sigma I)$ are i.i.d. noise terms for some $\sigma > 0$. We can estimate the A as following:

$$\hat{\mathbf{A}} = \sum_{t=2}^{m} \mathbf{s}_t \mathbf{s}_{t-1}^T \left(\sum_{t=2}^{m} \mathbf{s}_{t-1} \mathbf{s}_{t-1}^T \right)^+,$$

Following the treatment in(Box, G.E., Tiao, G.C., 1977)and (D'Aspremont, 2011), we calculate the predictability of the portfolio (λ) as

$$\lambda = \frac{n^T \hat{A} G \hat{A}^T n}{n^T G n},$$

Where n is our portfolio vector and G is the stationary covariance matrix of process s_t .Assuming that the noise terms in equation () are i.i.d. with $\mathbf{W}_t \sim N(0, \sigma I)$ for some $\sigma > 0$, we obtain the following estimate for σ using $\hat{\mathbf{A}}$.

$$\hat{\sigma} = \sqrt{\frac{1}{N(m-1)} \sum_{t=2}^{m} \left\| \mathbf{s}_t - \hat{\mathbf{A}}\mathbf{s}_{t-1} \right\|^2}.$$

Where m is the length of our observation. For calculating the μ we used sample mean.

$W(t)$ is a Wiener process. The solution of (2) is given as

$$P(t) = P(0)e^{-\lambda t} + \mu(1 - e^{-\lambda t}) + \int_{s=0}^{t} \sigma e^{-\lambda(t-s)}\, dW(s). \qquad (3)$$

Note that this equation tells us that λ must be positive, while $P(0), \mu$ and $P(t)$ could be either positive, or negative. Without the loss of generality we assume that σ is also positive. The price of the portfolio $P(t)$ has very nice properties, e.g. its mean is given as

$$\mathbf{E}\{P(t)\} = P(0)e^{-\lambda t} + \mu(1 - e^{-\lambda t}), \qquad (4)$$

entailing that the portfolio tends to return to the stationary mean as time tends to infinity.In the far future $P(t)$ becomes a Gaussian random variable

$$\lim_{t\to\infty} P(t) \sim \mathrm{N}\left(\mu, \sqrt{\frac{\sigma^2}{2\lambda}}\right). \qquad (5)$$

Thus, if λ is large, μ of $\mathbf{E}\{P(t)\}$ is quickly quickly growing and the stationary deviation is small. Hence, the portfolio quickly reverts to μ and its value can be easily predicted. As a result, for technical investors, such type of mean reverting portfolios $\underline{n}_{opt}$ should be sought which maximize the corresponding λ value .

However, maximizing parameter λ is not sufficient under most circumstances. Let us assume that (λ, μ) parameter pairs could be estimated and they are known for the investor. If $P(0) \approx \mu,$ it means that λ could be arbitrary large and the expected profit equals zero. Thus, maximizing solely the λ parameter does not necessary yield the best portfolio. Letus rewrite (3) in the following form:

$$P(t+\Delta t) = P(t)e^{-\lambda\Delta t} + \mu(1-e^{-\lambda\Delta t}) + \nu, \qquad (6)$$

where $\nu = \int_{s=t}^{t+\Delta t} \sigma e^{-\lambda(t+\Delta t - s)}\, dW(s),$, which is a zero mean Gaussian noise. In this way, $P(t)$ could be simply written as

$$P(t+\Delta t) = a \cdot P(t) + b + \nu, \qquad (7)$$

where $a = e^{-\lambda\Delta t}$, $b = \mu(1-e^{-\lambda\Delta t})$. We return to the linear model later. Thus, instead of maximizing parameter λ we rather minimize the probability of negative return (loss) in order to provide gain with minimal risk in probabilistic term.

3 Portfolio selection by minimizing the loss probability

We start from (3) corresponding to the Wiener process $W(s)$ and we want to find the parameters which yield the maximum probability for positive profit (positive refers to the case when it is larger than the minimum expected). That is, the loss (compared to the minimum expected profit) is minimized in probability.

We assume that parameters μ, λ could be derived for all possible portfolios and the investor defines a time frame t which limits the holding time of the portfolio and an expected return r. The expected return should come from an alternative investment which has similar properties, e.g. the same risk. Without the loss of generality we assume that the investor opens a position (either short, or long) at $t=0$ at the price of $P(0)$ and observes the price movements $P(t)$. Note that for the sake of generality, parameters μ and $P(0)$ could be either positive, or negative and they could also have opposite signs. Negative $P(0)$ means: opening a long position yields cash, opening a short position costs money. Similarly, negative $P(t)$ means: closing the long position needs cash, closing a short position results in cash.

It is clear that if $\mu > P(0),$ then the investor should take a long position, independently of the sign of $P(0)$: it is expected that the price of the portfolio will go up (returning to the mean) yielding profit. Otherwise, if $\mu < P(0),$ the investor should take a short position, expecting positive profit independently of the sign of $P(0)$. If $\mu \approx P(0)$ then the investor should not take any position.

If there is an open position, the investor can either win, or loose, depending on the $P(t)$ value at the time of closing its position.Depending on the signs of μ and $P(0)$ and their relation, six different cases could happen. These are depicted in Figure 1. In the odd cases (left hand side coloumn), μ is greater than $P(0)$, thus long position should be taken. On the contrary, even cases (right hand side coloumn), μ is smaller than $P(0)$, thus short positions should be opened at $t=0$. Please note that Case 1 can be considered as minus one times the portfolio of Case 6 (taking a long position in a positive portfolio is the same as taking a short position in the negative portfolio). Similarly, Case 2 can be transformed into Case 5. Case 3 and Case 4 could be also replaced by each other. Finally, we can say that one coloumn describes all possible events.

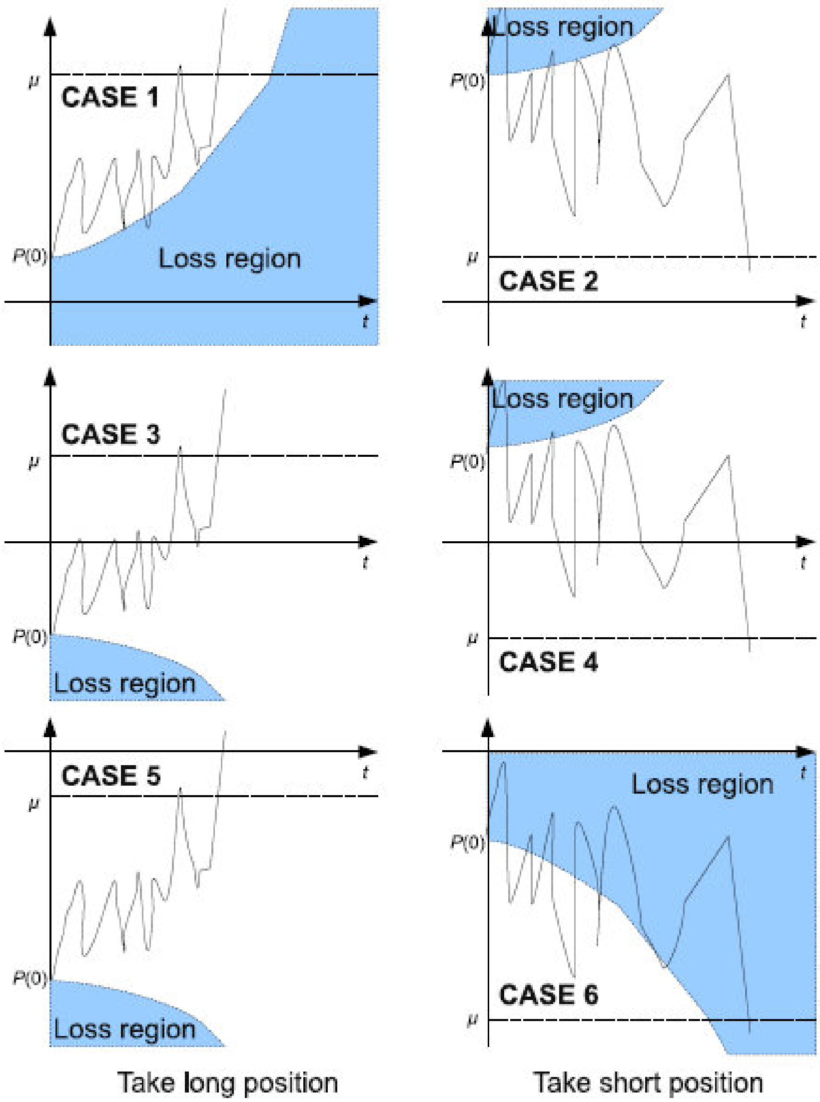

Figure 1: Six different cases for *P*(0) and μ values.

First of all, $P(t)$ must be compared against $P(0)(1+r)^t$, , that is, holding the portfolio for a t period is expected to yield interest also, which is represented in this formula. The larger r is, the larger the interest becomes, and the interest grows exponentially with time t. When the long position was opened at $t=0$ $(\mu > P(0))$, then $P(t) < P(0)(1+r)^t$ describes the situation of loss, the prompt value of the portfolio is smaller than the expected wealth. On the other hand, when short position was opened at time $t=0$ $(\mu < P(0))$, then $P(t) > P(0)(1+r)^t$ describes the event of loss. Figure 1 shows the region of loss in blue colour.

The probability of loss (P_L) could be expressed as the probability of falling inthe blue region. That is, if the purchased portfolio worths less than a financial instrument with the expected return r for the given time t,

$$P_L = \begin{cases} \mathbf{P}\left\{P(t) < (1+r)^t P(0)\right\}, & \text{if } \mu > \text{P}(0), \text{long position is taken,} \\ \mathbf{P}\left\{P(t) > (1+r)^t P(0)\right\}, & \text{if } \mu < \text{P}(0), \text{short position is taken.} \end{cases}$$

These two equations could be easily combined into one:

$$P_L = \mathbf{P}\left\{(\mu - P(0))\left(P(t) - (1+r)^t P(0)\right) < 0\right\}. \qquad (8)$$

Clearly, investors should minimize (8), i.e. the probability of loss.

3.1 Mean reverting portfolios

In mean reverting portfolios, $P(t)$ must be substituted from (3) into (8). Thus, one gets

$$P_L = \mathbf{P}\left\{(\mu - P(0))\left(P(0)e^{-\lambda t} + \mu(1-e^{-\lambda t}) + \frac{\sigma}{\sqrt{2\lambda}}\nu - P(0)(1+r)^t\right) < 0\right\},$$

where ν is a standard Gaussian process ν~$N(0,1)$. Since there is no sense of trading, if μ=$P(0)$ (the long run expected value of the portfolio equals the start price), we can assume that the equality does not hold. Rewriting it, one gets:

$$P_L = \mathbf{P}\left\{\nu \lessgtr \frac{\sqrt{2\lambda}}{\sigma\left(1-e^{-2\lambda t}\right)}\left(P(0)\left((1+r)^t - e^{-\lambda t}\right) - \mu\left(1-e^{-\lambda t}\right)\right)\right\},$$

where the upper inequality refers to the case when μ–$P(0)$>0, or μ>$P(0)$ and the lower one holds for the opposite case. The equation is easily computable, since ν follows a standard normal distribution. That is, using the error function (erf(.)) we can express the above probability, as

$$P_L = \frac{1}{2} \pm \frac{1}{2} erf\left(\frac{P(0)\sqrt{\lambda}}{\sigma\left(1-e^{-2\lambda t}\right)}\left((1+r)^t - e^{-\lambda t} - \frac{\mu}{P(0)}(1-e^{-\lambda t})\right)\right), \qquad (9)$$

where the ± sign describes both cases (μ>$P(0)$ and μ<$P(0)$), and

$$erf(x) = \frac{2}{\sqrt{\Pi}}\int_0^x e^{-t^2}dt.$$

The results show us how the probability of loss can be computed, if all the parameters of (9) is known ($\lambda, \sigma, r, t, \mu, P(0)$ must be determined for the calculation). Note that for $t = 0$ the loss probability (9) equals 50% on a very tiny time scale. It can be easily accepted, since we have Brownian motion in the system and the probabilities of moving up, or down are equally 50%. As $t \to \infty, P_L \to 1,$, if μ and $P(0)$ has the same sign, and $|P(0)| < |\mu|$, and $P_L \to 0,$, otherwise. That is, in the far future, we have one probability of loss, if $P(0)$ is closer to zero, than μ and they are in the same half of the plane (Case 1 and Case 6 in cases). The explanation is simple: we have exponential expectation in the profit $(\sim (1+r)^t)$, while the price of the portfolio tends to its finite mean (μ). That is, after a sufficiently long time period, the expectation will exceed the mean with probability one, even though there is a Gaussian effect in the system.

On the other hand, in the far future, we have zero probability of loss, if $P(0)$ and μ have different signs (Case 3 and Case 4 in Figure 1), or if they do, μ is closer to zero than $P(0)$ (Case 2…Case 5 in Figure 1). This is more interesting from the investor point of view. The reason for having such a lucky scenario is that the price of the portfolio tends to a constant value μ, while the money we got for the long/short position (note that in all four cases we receive cash) yields an exponentially growing profit (with the alternative investment). The more time we wait, the more profit we have. In the meantime, the price of the portfolio tends to a finite value with smaller variance. That is, with probability one, we can pay the price of closing our (either long or short) position. The reader might be surprised having probability one for winning in the long run, however, we should not forget that

- short positions are admitted for only a limited time frame (the short position must be closed after a pre-defined period),
- our model is quite simplified, since the mean reverting model does not include exponential growth of μ. The price tends to a constant value with lower variance.

Extending the mean reverting model without this constant nature is not considered in this paper and will be investigated in the future.

One might choose a trading strategy that (9) is minimized. As explained earlier, it makes sense only when μ and $P(0)$ has the same sign, and $|P(0)|<|\mu|$. For the sake of minimization at least one of the parameters must be released. We will investigate two cases: (1) the interest rate (r) is variable, and (2) the portfolio holding time (t) is variable. In both cases we make use of the fact that the error function is a monotonously increasing function, that is, minimizing (9) is the same as minimizing/maximizing the argument of the *erf* function. For the sake of milder notations, the argument of the erf(.) function will be denoted by $f(P(0),\lambda,\sigma,r,t,\mu)$ in the sequel.

The first case can be easily analyzed, if the interest rate of the alternative investment (r) is an open parameter, it should be set to zero, since $r=0$ yields the lowest possible value in (9).[1] As r deviates more from zero, the probability of loss becomes larger. There is no sense to optimize the value of r. Thus, only fixed r is considered in the following discussion.

If r is fixed, the investor might be interested to find an optimal time frame t_{opt} for which the portfolio should be held. For the sake of minimization

$$t_{opt} = \arg\underset{t}{\overset{\min}{\max}} f(P(0),\lambda,\sigma,r,t,\mu),$$

where $f(.)$ is the argument of the *erf* function, as given in (9). Standard calculations can be used to get the optimal $t.t.$ First the derivative of $f(P(0),\lambda,\sigma,r,t,\mu)$ is computed.

$$\frac{\partial}{\partial t} f()\Big|_{t=t_{opt}} = \frac{P(0)\sqrt{\lambda}}{\sigma}.$$

$$\frac{\left(\ln(1+r)(1+r)^{t_{opt}} + \lambda e^{-\lambda t_{opt}}\left(1-\frac{\mu}{P(0)}\right)\right)\left(1-e^{-2\lambda t_{opt}}\right) - \left((1+r)^{t_{opt}} - e^{-\lambda t_{opt}} - \frac{\mu}{P(0)}\left(1-e^{-\lambda t_{opt}}\right)\right)2\lambda e^{-t_{opt}}}{\left(1-e^{-2\lambda t_{opt}}\right)^2} = 0. \quad (10)$$

After some further calculations it finally yields:

$$t_{opt} = \frac{1}{\lambda}\ln\left(-1-\lambda\frac{1-\frac{\mu}{P(0)}}{\ln(1+\mathrm{r})} + \sqrt{1+6\lambda\frac{1-\frac{\mu}{P(0)}}{\ln(1+\mathrm{r})} + \lambda^2\left(\frac{1-\frac{\mu}{P(0)}}{\ln(1+\mathrm{r})}\right)^2}\right). (11)$$

Note that the argument of the second logarithm: $\mu / P(0)$ must be greater than 1, otherwise the logarithm becomes negative. The same has been supposed previously: only Case 1 and Case 6 from cases are of

[1] The reader should note that $r=0$ says that there is no interest rate connected to alternative investments. Obviously, it is not practical.

interest. The optimal timeframe for the portfolio thus can be computed if all the parameters $(\lambda, \mu, P(0), r)$ are available. Using this t_{opt}, (9) can be evaluated.

The reader should note that one former statement is also proven by (10): the left hand side of partial derivative is positive for all t values. Thus, if $1-\frac{\mu}{P(0)}$ is positive, then the whole expression is also positive for all t values. In other words, there is no extremal value of $f(.)$ function. Vica versa, if $1-\frac{\mu}{P(0)}$ is negative, we can find the optimum by finding the time t which yields zero in (10). Only Case 1 and Case 6 from cases yields supervalues.

As an example, Figure 2 shows the case, where $P(0)=1, \mu=2, \lambda=10, r=0.1, \sigma=1$. Both axes are given in logarithmic scale. As one can compute, (11) yields t_{opt} = 1.4527. The figure shows well how the curve behaves, starting from 0.5 and tending to 1 yielding a global minimum at the described point.

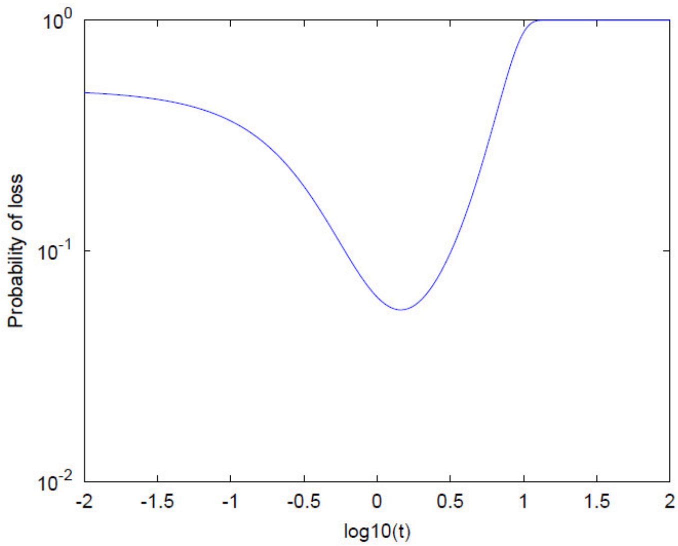

Figure 2: An example with the parameter set $P(0)=1, \mu=2, \lambda=10, r=0.1, \sigma=1$.

One must not forget that the minimum exists only when μ and $P(0)$ has the same sign, and $|P(0)|<|\mu|$. In all other cases, the longer t is, the lower the probability of loss P_L Usually this is limited by some external factors, short positions must be closed after a given amount of time (denoted by t_{close}).

The process of portfolio selection is depicted in Figure 3. First, all possible portfolios must be identified. As a next step, for each portfolio, the parameters of the portfolio must be estimated. Since (11) could be computed for portfolios, where $\mu / P(0)$ is greater 1. than only those portfolios are considered in Step 4 where this assumption holds. This is clearly only a subset of all possible portfolios. This subset is optimized with an exhaustive search. The best loss probability and t_{opt}, value must be remembered.

The rest of the portfolios are considered in Step 5. The probability of loss values are computed based on the t_{opt}, value which is related to the best portfolio from the subset of Step 4. The computed probability of loss value is compared against the best probability of loss of thebest portfolio from the subset of Step 4. If a better portfolio is found, then it must be remembered.

Finally, the best portfolio must be chosen, where the „best" means the one with the lowest probability of loss.

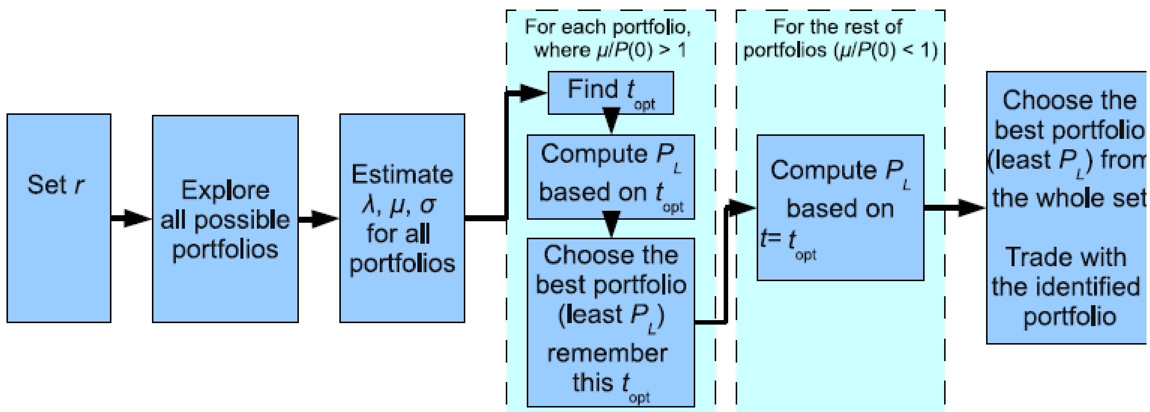

Figure 3: The process of portfolio selection based on loss probability minimization

4 Numerical results

Our new „loss-probability-minimization" strategy has been tested on FOREX data series. For the sake of comparable performance, we tested the lambda maximization strategy against the proposed method. In our comparisons we tried to be fair: we did not apply any sophistication, or plug-in that to optimize the models.

We have one model, the mean reverting portfolio model but developed another objective function beside maximizing parameter lambda, i.e. minimizing the probability of loss. The two algorithms are given in Table 1. The decision limits are given in Table 2.

Table 1: The algorithms used for obtaining numerical results

Nick	Model	Objective
MaxLambda	Mean Reversion	Maximizing the lambda
MinPrLoss	Mean Reversion	Minimizing the Probability of Loss

Here we used a very simple trading system. First we take the best portfolio, based on the objective function. We immediately initiate the trading position based on the portfolio's weights (positive weight means buy – long position, negative weight means sell – short position), if the price is closer to zero than $P(0)$ ($|P(t)|\leq|P(0)|$). If it is not, we wait for getting closer to zero (hitting $P(0)$), meanwhile the portfolio selection algorithm is rerun. If another portfolio is chosen, a new $P(0)$ is defined (according to the actual price of the selected portfolio) and we try to open the position at a price closer to zero than the new $P(0)$. This process is run until the position is opened.

The position is closed based on the mean (μ) and t_{opt}. If the mean is hit ($|P(t)|>|P(0)|$). then the position is closed after the first positive peak for both methods. For the loss probability minimization algorithm, if t_{opt} has elapsed, and we have not yet hit the mean, than $P(t_{opt})$ must be hit with the same rule (after the first peak, we close the position). After 10 days of the position opening, if the price is still in a valley, the position will be closed. For the StopLoss versions, the StopLoss limit is set as $P(0)-10(\mu-P(0))=11P(0)-10\mu$. This setting yields a Risk/Reward ratio equal to 10.

As soon as the position is closed, another portfolio is going to be selected, and the process starts from the beginning.

Table 2: Decision limits for the two algorithms

	MinPrLoss		MaxLambda	
	No SL	StopLoss	No SL	StopLoss
Entry Point	$\|P(t)\| \leq \|P(0)\|$			
Take Profit	Mean (μ), or $P(topt)$		Mean (μ)	
StopLoss (SL)	$P(topt)$, 10 days	$11P(0)-10\mu$	–	$11P(0)-10\mu$

We used four assets (EURUSD, GBPUSD, AUDUSD, NZDUSD) for the FOREX operations. The sparsity constraint was set to three (maximum 3 of 4 assets were chosen for each portfolio), the trading frequency was one day (operations were taken once per day). The testing period of the algorithms was between 2009 and 2012. The initial (virtual) deposit was 1000 USD and no leverage was used (the leverage ratio was 1:1).

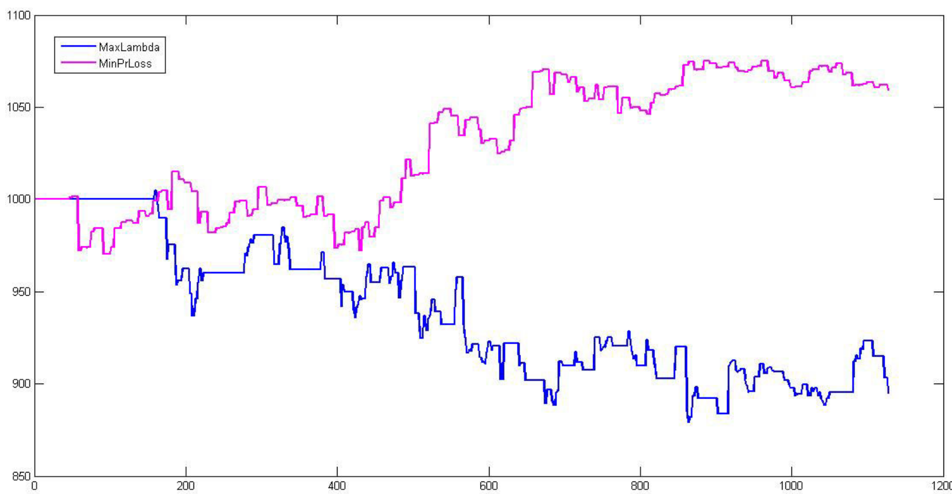

Figure 4: The performance of the two algorithms – Equity in case of existing stop loss

Figure 4 shows the performance of the two algorithms. One can see, that the strategy "minimizing probability of loss" strikingly outperforms the traditional strategy which focuses on maximizing lambda. The poor performance of the latter one is due to the fact that there are many positions closed in loss, since the stop loss limit is hit. For fair comparison we also show the performance, when stop loss limit is switched off.

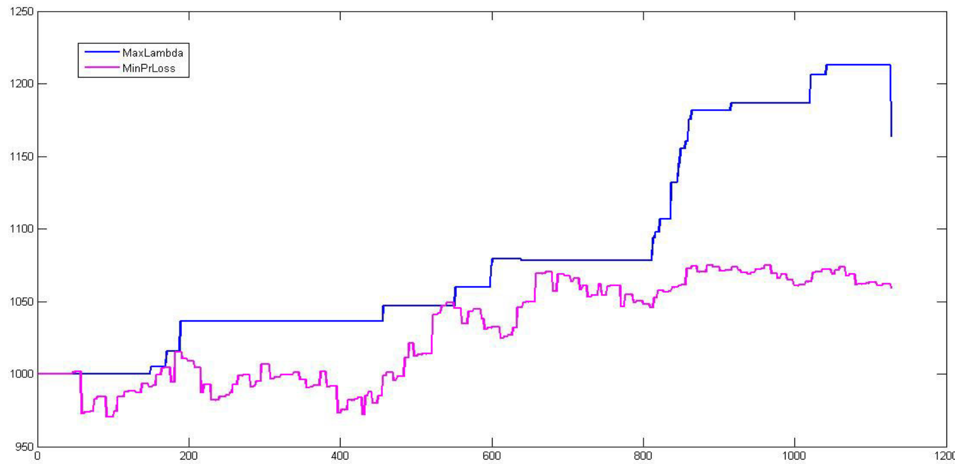

Figure 5: The performance of the two algorithms – Equity in case of absence of stop loss

Although the strategy “maximizing lambda” is better without stop loss in Figure 5, one should not forget that the lambda maximization algorithm yields worse balance in the flat periods, when the equity remains constant. That is, if there is a leverage applied, this strategy could easily yield bankruptcy. Meanwhile, the algorithm based on the minimization of the loss probability presented the maximum dropdown of 4.3%. That is, even with a leverage ratio of *20*:1, it will avoid bankcruptcy.

5 Conclusions

In this paper we have introduced a novel portfolio selection, based on minimizing the loss probability under the assumption of mean reverting property. After identifying the the corresponding parameters of the mean reverting process, one can compute probability of making negative profit which the so-called loss probability. Since this computation can be carried out for a large number of portfolios, we can choose the one which has the lowest loss probability. The numerical results obtained on FOREX data have been demonstrated that higher profit can be achieved by the new strategy than simply maximizing the mean reverting parameter λ.

However, the complexity of the method can become overwhelming due to the exhaustive search in the space of all possible portfolios. As a future work, the authors are interested in finding better strategies, where the complexity is reduced.

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